DYNAMICS OF A FRICTIONAL SYSTEM, ACCOUNTING FOR HEREDITARY-TYPE FRICTION AND THE MOBILITY OF THE VIBRATION LIMITER

Leonid A. Igumnov, Vladimir S. Metrikin[*], Mikhail V. Grigoryev

Research Institute for Mechanics of the National Research Lobachevski State University of Nizhniy Novgorod,

603109, Nizhny Novgorod, Gagarina Avenue 23, Russian Federation

*e-mail: v.s.metrikin@mail.ru

Abstract. Dynamics of a frictional system consisting of a rough body situated on a rough belt moving at a constant velocity is studied. The vibrations are limited by an elastic obstacle. The Coulomb-Hammonton dry friction characteristic is chosen, according to the hypothesis of Ishlinskiy and Kragelskiy, in the form of hereditary-type friction, where the coefficient of friction of relative rest (CFRR) is a monotone non-decreasing continuous functionof the time of relative rest at the previous analogous time interval. A mathematical model and the structure of its phase space are presented, as well as the equations of point maps of the Poincare surface and the results of studying the dynamic characteristics of the parameters of the system (velocity of the belt, type of the functional relation of CFRR, rigidity of the elastic obstacle, etc.). A software product has been developed which makes it possible to find complex periodic motion regimes, as well as to calculate bifurcation diagrams used for determining the main variations of the motion regime from periodic to chaotic (the period doubling scenario).

Keywords: frictional system, vibrations, elastic obstacle, dry friction, hypothesis of Ishlinskiy and Kragelskiy

1. Introduction

The present paper is a direct continuation of a series of works (see [1]-[2] and the references thereof) and some others [3-10], studying dynamic characteristics of vibrational systems with friction, accounting for its hereditary type, or without it. As is shown in the above works, accounting for its heredity gives rise to the nonlinear dynamics and the motions that have not been earlier observed in the mathematical models of such systems involving the classical description of friction forces. However, self-excited vibrations provoked by friction are of paramountimportance in the contemporary applied engineering. Some of the examples of mechanisms and technical devices where the problem of frictional self-oscillations plays an important part are drilling rigs [11], braking units [12-19], friction bearings [20-23], friction coupling mechanisms [24-33], self-oscillations in machining tools resulting from functional displacements of their working parts and from deep drilling processes [34]. The paper pays special attention to developing and analyzing the dynamics of the mathematical model, taking account of the interaction between the end effector and the medium being machined.The specific feature of the investigation approach is that a point map is formed not in a classical way (mapping a Poincare surface into itself) but based on the times of relative rest of the vibration system, which significantly simplifies both the process of point mapping itself and

its detailed study. The presence of floating boundaries of plates of sliding motions necessitated the development of an original approach to point mapping and to the interpretation of the results obtained. Using the developed investigation methodology and the created software product, the structure of the phase portrait of the mathematical model has been studied as a function of the characteristics of friction, sliding and static friction forces, as well as of the type and position of the limiter. Based on the character of changing of the bifurcation diagrams, the main laws of the process of change of the motion regimes (periodic regimes of arbitrarily complex motion and a possible transfer to chaos via the period-doubling process) when changing the parameters of the vibrational system have been determined.

2. Mathematical model

Consider frictional self-oscillations of a system (Fig. 1a) consisting of a body of mass m_1, attached to a fixed obstacle with spring stiffness c_1, situated on a rough belt moving at a constant velocity V, in the assumption that the coefficient of sliding friction between the body and the moving belt is constant, and CFRR f is a monotonous increasing function of time $t = t_k$ of relative rest of the body and the belt at a previous time (Fig. 1b).The motion of the body is limited by an elastic obstacle with stiffness c_2, situated at distance $"a"$ from the body.
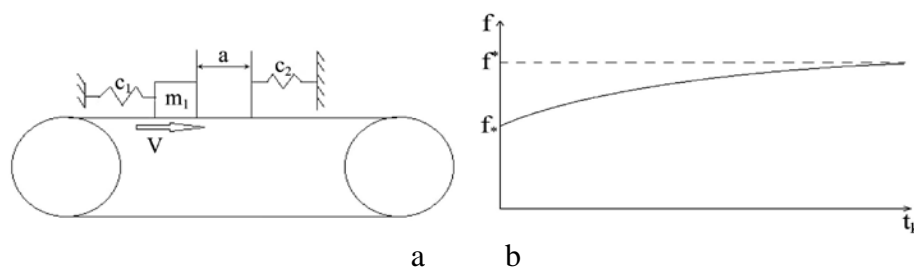

a   b
Fig. 1. A physical model of the system

Let the duration of the $k-$th interval of a prolonged contact of body m_1 with the belt be equal to t_k . The dry friction force within this interval is higher than the force of the spring, $f(t_k)P_1 \ge |c_1 x_1|$, and the body moves together with the belt according to equation $\dfrac{dx_1}{dt} = V$, where coordinate x_1 coincides with deformation of the spring c_1, P_1 is force of the normal pressure. Then the equation of motion of body m_1 at the velocity of the belt can be written as

$$\frac{dx_1}{dt} = V, |c_1 x_1| \le f(t_k)P_1 , x_1 \le a . \tag{1}$$

At the stages of sliding of body m_1, the equation of motion will have the following form:

$$m_1 \frac{d^2 x_1}{dt^2} + c_1 x_1 = -f_* P_1 sign\left(\frac{dx_1}{dt} - V\right), \frac{dx_1}{dt} \ne V, x_1 \le a . \tag{2}$$

When the body reaches the elastic wall at the stage of relative rest of the belt and of the body, the motion of the latter will be governed by the following differential equation:

$$\frac{dx_1}{dt} = V, |c_1 x_1 + c_2 (x_2 - a)| \le f(t_k)P_1, x_1 > a , \tag{3}$$

and at the sliding stages by equation

$$m_1\ddot{x}_1 + c_1x_1 + c_2(x_1 - a) = -f_*P_1 sign(\dot{x}_1 - V), \dot{x}_1 \neq V, x_1 > a. \tag{4}$$

Introducing dimensionless time $\tau = \sqrt{c_1/m_1}\,t$, coordinate $\xi = c_1x_1/(f_*P_1)$ and parameters $\theta = \sqrt{m_1c_1}V/(f_*P_1)$ (velocity of the belt), $C = c_2/c_1$ (ratio of the stiffnesses of the springs, $A = c_1a/(f_*P_1)$ (distance between the body and the elastic obstacle),equations (1-4) can be rewritten in the form of the following equation set:

$$\begin{cases} \dot{\xi} = \theta, |\xi| \leq 1 + \varepsilon_k, \xi \leq A, \\ \ddot{\xi} + \xi = -sign(\dot{\xi} - \theta), \dot{\xi} \neq \theta, \xi \leq A, \\ \dot{\xi} = \theta, |\xi + C(\xi - A)| \leq 1 + \varepsilon_k, \xi > A, \\ \ddot{\xi} + (1+C)\xi + CA = -sign(\dot{\xi} - \theta), \dot{\xi} \neq \theta, \xi > A, \end{cases} \tag{5}$$

Where $\varepsilon(\tau) = (f(\tau) - f_*)/f_*$ is dimensionless functional characteristic of the coefficient of friction of relative rest, and $\varepsilon_k = \varepsilon(\tau_k)$.

3. The phase space structure

The phase space of the system is plane $(\xi, \dot{\xi})$ divided by straight line $L(\dot{\xi} = \theta)$ into three subspaces, $F_+(\dot{\xi} > \theta), F_0(\dot{\xi} = \theta), F_-(\dot{\xi} < \theta)$, within each of which the behavior of the phase trajectories is described by different systems of differential equations. It is to be noted thatif $A > 1$, as is shown in Fig. 2.
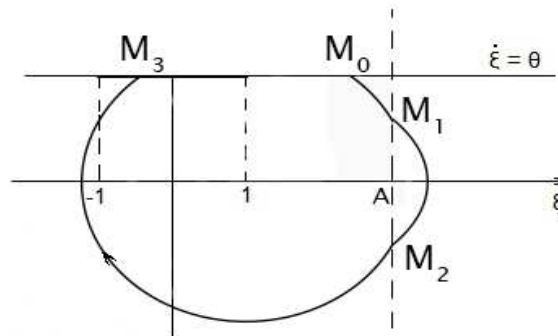

Fig. 2. A phase space

Then it can be shown that the phase trajectories in subspaces F_+ and F_- for $\xi \leq A$ have the form of circumferences with the centres $(-1,0)$ and $(1,0)$, respectively. Segment $S(-1 \leq \xi \leq 1, \dot{\xi} = \theta)$ of the junction of the phase trajectories lies on half-line $l(\xi \geq -1, \dot{\xi} = \theta)$, on which the phase trajectory has the form of straight line $\dot{\xi} = \theta$, the motion going on along this line as long as the conditions of the first equation of system (5) are satisfied. As the physical system considered involves friction, the representing point will eventually be on segment S of subspace F_0. Thus, the dynamics of the system can be analyzed using the method of point mapping of half-line l into itself [35-36].

4. The point mapping method. Constructing Poincare function

The application of the point mapping method will be illustrated by consticting Pointcare function for finding and studying the issue of the existance of stable periodic motions with punching an elastic obstacle. Let $M_0(1 + \varepsilon_k, \theta)$ be initial point of transition from subspace F_0

into subspace F_- of a phase trajectory after its another motion along half-line l for τ_k. The unknown coordinates of points $M_1(A, \dot{\xi}_1), M_2(A, \dot{\xi}_2), M_3(\xi_3, \theta) \in S$ have the following form:

$$\dot{\xi}_1 = \sqrt{\theta^2 + (1 + \varepsilon_k)^2 - (A-1)^2}, \dot{\xi}_2 = -\dot{\xi}_1, -1 < \xi_3 = -\varepsilon_k < 0. \tag{6}$$

Making use of relation (6), one can correlate the two successive times of relative rest of the body as

$$\psi(\tau_{k+1}) = \varphi(\tau_k), \psi(\tau) = \theta\tau - \varepsilon(\tau), \varphi(\tau) = 1 + \varepsilon(\tau). \tag{7}$$

In this way, Poincare function (7) is obtained, but not in its classical form of a functional relationship of two successive coordinate points on the Poincare surface, but using an implicit relationship between two successive times of relative rest of the body with the belt, which significantly simplifies further analysis of the properties of Poincare function. Thus, choosing a continuous strictly increasing function, describing the functional relationship of the coefficient of friction of relative rest in dynamics, in the form of

$$\varepsilon(\tau) = \varepsilon_*(1 - \exp(-\delta\tau)), \varepsilon_* = (f^* - f_*)/f_* > 0, \delta > 0 \tag{8}$$

and using the last inequality in equations (6), one can find an equation defining 'coordinate' τ^* of the fixed point of Poincare function $\psi(\tau^*) = \varphi(\tau^*)$, corresponding to stable half-turnaround periodic motion of the body with one long period of relative rest of the body and one instance of compression of the elastic obstacle, as

$$g_1(\tau^*) = g_2(\tau^*), g_1(\tau^*) = 2\varepsilon_* + 1 - \theta\tau^*, g_2(\tau^*) = 2\varepsilon_* \exp(-\delta\tau^*). \tag{9}$$

Figure 3 depicts the form of functions $g_1(\tau), g_2(\tau)$, from which it follows that equation (9) has a unique solution, for which one can readily determine the interval where it can be found, $\tau^* \in (\theta^{-1}, (1 + 2\varepsilon_*)/\theta)$. The stability in small of a fixed point is determined by the condition on the parameters of the system of the form

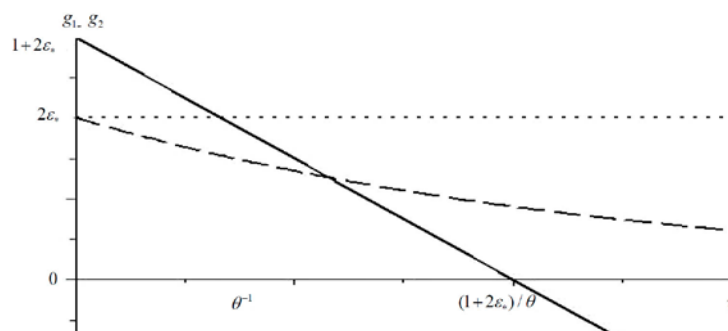$$\theta > 2\varepsilon_* \delta \exp(-\delta\tau^*). \tag{10}$$

Fig. 3. Function graphs $g_1(\tau^*)$ (solid line) and $g_2(\tau^*)$ (dash line)

5. Results of numerical experiments

To analyze the complex nonlinear dynamics, a software product has been developed in the mediumMS Visual Studio 12 ®inC++, which makes it possible to find the main alterations in the motion regimes of a body, based on bifurcation diagrams and phase portraits.

Figure 4 depicting a bifurcation diagram for parameter θ, vividly illustrates the alteration of the periodic motion regimes: the increasing velocity gives rise to periodic motions with the increasing number of times of relative rest of the body with the belt (the 'period doubling' process), leading, as is known [36-37], to chaos.
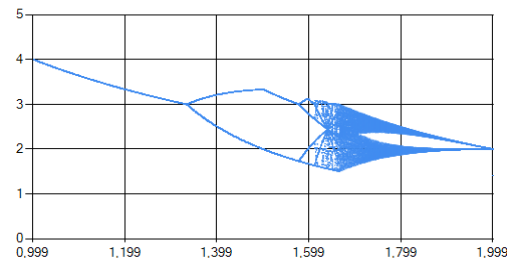
Fig. 4. Bifurcation diagram for parameter θ

6. Conclusions

1. A mathematical model of a vibrational system with hereditary-type friction, accounting for the elasticity of the vibration limiter, has been presented.

2. A description of the phase space has been introduced, from which it was concluded that the dynamic system is a system with a variable structure.

3. The nonlinear dynamics has been analyzed numerically-analytically, using a modified mathematical apparatus of the point mapping method.

4. To analyze simplest periodic motions, an analytical form of Poincare function has been introduced, and the existence of a unique fixed point corresponding to periodic motion with one instance of prolonged relative rest and one instance of compression of the elastic obstacle has been proved. The presented bifurcation diagram once again corroborated the known scenario of the development of chaotic motion regimes.

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