



Mathematical Modeling of Nonlinear Dynamic Deformation and Failure of Metal-Plastic Shells of Revolution

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Abstract. A technique for numerical analysis of nonlinear dynamic deformation and progressive failure of multi-layered metal-plastic shells of revolution is developed with account for their strain-rate dependent strength characteristics. The geometric dependencies are formulated on the basis of quadratic version of the nonlinear theory of elasticity. The relationship between stress and strain tensors in a composite macrolayer is based of Hooke's law for an orthotropic body combined with the theory of effective modules. The process of progressive layer-by-layer failure is described in the framework of the degradation model of stiffness characteristics. The strain rate dependent stiffness and strength characteristics are accounted for. An energetically consistent system of equations of motion is constructed using the principle of possible displacements. A numerical method for solving the problem is based on an explicit variational-difference scheme. The proposed technique was verified on the problem of unsteady deformation of a cylindrical shell subjected to pulse pressure.

Keywords: Composite materials · Shells of revolution · Strength · Failure · Numerical methods · Explosive loading

Due to their effective energy absorption, increased crack-resistance and non-splintering character of possible failure, composite materials are widely used in constructing protective structures subjected to intense pulse loadings. In this connection, experimental and theoretical studies of strain rate effect of composite materials on their strength and stiffness characteristics appear to be highly topical since the material of structural elements subjected to pulse loading exhibits stress-strain behavior with a high degree of variability over time. In a number of experimental and theoretical works [1–3], the strain rate dependence of elastic and strength characteristics of composite materials is noted. However, the determination of the parameters characterizing the properties of composite materials at high strain rates is associated with certain difficulties due to the necessity to measure the effect of dynamic pressure on a specimen as a function of time.

The paper presents analytical results of the strain rate effect on dynamic behavior and progressive failure of shells of revolution made of hybrid metal-plastic materials.

We considered both homogeneous glass-fiber-reinforced plastic shells formed by double alternating winding of spiral and annular layers with a thickness ratio of 1:1, and inhomogeneous ones fabricated by spiral cross winding of unidirectional glass-fiber-reinforced plastic (according to the scheme of reinforced homogeneous shells) on a steel cylindrical mandrel made of mild steel.

Since the shells of revolution made from composite materials are inhomogeneous, have low shear stiffness and, in some cases, are rather thick, thus, in order to describe their stress-strain state, it is necessary to use nonclassical shell theories [4].

Geometrical dependencies are constructed using the relations of the simplest quadratic version of the nonlinear elasticity theory [4].

The stress-strain tensors in a homogeneous composite macrolayer are related by Hooke's law for an orthotropic body in combination with the theory of effective moduli [4]. The process of progressive layer-by-layer damage of layered shells of revolution is described in the framework of the model of degradation of their stiffness characteristics [5]. And furthermore, the strain-rate dependent strength characteristics of composite materials are taken into account. In particular, for unidirectional fiberglass, the material stiffness and strength characteristics can be described by the regression function [3]

$$F(\dot{\epsilon}) = \alpha + \beta\dot{\epsilon}^\gamma \quad (1)$$

where F and $\dot{\epsilon}$ are strength characteristics and strain rate; α , β , γ are experimentally determined material constants. The constitutive relations in the isotropic steel layer of the shell are formulated on the basis of the differential theory of plasticity with linear hardening [4].

In order to derive the equations of motion of an inhomogeneous shell of revolution, the principle of virtual displacements is used [6].

The equations derived are universal enough, because they allow one to describe nonlinear nonstationary deformation processes and to estimate the limiting deformability and dynamic strength of two-layered metal-plastic shells of revolution, and their geometrical and structural parameters changing over a wide range.

The numerical method for solving the problem formulated was based on the explicit variational-difference scheme [4, 7]. The calculations were carried out on a "Lobachevsky" supercomputer.

A numerical analysis of the effect of the strain rate on the dynamic strength of two-layered metal-plastic cylindrical shells was considered on the problem of their deformation under the action of a pressure pulse caused by a blast in a center of a shell of an explosion charge (EC), which describes the pressure profile in the incident shock wave by an empirical relation [8]. The physicommechanical characteristics of unidirectional glass-fiber-reinforced plastic shells were determined from the results of quasistatic tests on ring and plane unidirectional specimens by standard techniques for identifying the corresponding stiffness and strength characteristics [9]. The strain rate dependence of the strength characteristics of glass-fiber-reinforced plastics is described by function (1).

The results obtained attest that the account of strain rate dependence of strength characteristics for all the reinforcement schemes considered lead to a qualitative difference in the character and size of failure zones of the binder and fibers and to a

significant increase in the load-carrying capacity of the shells compared with the calculations of constant strength characteristics.

Calculation models allowing one to analyze the processes of progressive failure of pulse-loaded metal-plastic cylindrical shells of revolution both with account of the strain rate dependence of their strength characteristics (dynamic model) and with constant characteristics (static model) have been proposed.

A comparative analysis of calculation results with experimental data testifies their better agreement in dynamic model. For various reinforcement schemes of composite macrolayer, the qualitative differences in character and size of failure zones calculated by both the methods were revealed.

The results obtained can be used in the design and evaluation of the dynamic strength of load-carrying elements.

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